AIAA JOURNAL
Vol. 48, No. 11, November 2010

Component Synthesis Method for Transient Response of Nonproportionally Damped Structures

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DOI: 10.2514/1.J050340

A component synthesis technique for calculating the transient response of nonproportionally damped structures is presented in this paper. The structure can be composed of nonproportionally damped components or proportionally damped components with different damping factors, and the components are joined by connecting elements at their boundaries. In the synthesis technique, the transient responses of the nonproportionally damped components are defined by a newly developed direct integration method, which keeps the analytical solution of the mode displacement superposition method for the proportionally damped structure with the interpolation of excitation forces as the input. The structural responses are obtained based on the geometric compatibility and the force equilibrium conditions of the connecting elements. The proposed method is most suitable for the recalculation of structural responses induced by modification of the connecting elements. Numerical examples are given to validate the correctness and to show the effectiveness of the proposed method.

Nomenclature

$\tilde{\boldsymbol{C}}$	=	damping matrix, $n \times n$
$\boldsymbol{C}_c$	=	connecting damping matrix
$\bar{c}_r$	=	rth modal damping, $\boldsymbol{\varphi}_r^T \tilde{\boldsymbol{C}} \boldsymbol{\varphi}_r$
$\boldsymbol{f}$	=	excitation force vector, $n \times 1$
$\bar{f}_r$	=	rth modal force, $\boldsymbol{\varphi}_r^T \boldsymbol{f}$
$\boldsymbol{K}$	=	stiffness matrix, $n \times n$
$\bar{\boldsymbol{K}}$	=	modal stiffness matrix, $\boldsymbol{\Phi}^T \boldsymbol{K} \boldsymbol{\Phi}$, $m \times m$
$\boldsymbol{K}_c$	=	connecting stiffness matrix
$\bar{k}_r$	=	rth modal stiffness, $\boldsymbol{\varphi}_r^T \boldsymbol{K} \boldsymbol{\varphi}_r$
$\boldsymbol{M}$	=	mass matrix, $n \times n$
$\bar{\boldsymbol{M}}$	=	modal mass matrix, $\boldsymbol{\Phi}^T \boldsymbol{M} \boldsymbol{\Phi}$, $m \times m$
$\bar{m}_r$	=	rth modal mass, $\boldsymbol{\varphi}_r^T \boldsymbol{M} \boldsymbol{\varphi}_r$
$\boldsymbol{q}$	=	modal coordinates vector, $m \times 1$
q_r	=	rth modal coordinate
t	=	time
$\boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}$	=	displacement, velocity, and acceleration vector, $n \times 1$
$\boldsymbol{x}_0, \dot{\boldsymbol{x}}_0$	=	initial displacement and velocity, $n \times 1$
$\boldsymbol{\Phi}$	=	real mode shape matrix, $(\boldsymbol{\varphi}_1 \quad \boldsymbol{\varphi}_2 \quad \cdots \quad \boldsymbol{\varphi}_m)$, $n \times m (m \le n)$
$\boldsymbol{\Phi}_j$	=	mode shapes associated with interface degrees of freedom
$\boldsymbol{\varphi}_r$	=	rth real mode shape vector

Subscripts

e	=	external forces and related responses
i	=	internal degrees of freedom
j	=	interface degrees of freedom, interface forces and related responses

Superscripts

α	=	substructure α
β	=	substructure β

I. Introduction

STRUCTURE design, modification, and structural dynamic analysis are closely concerned with the calculation of transient dynamic response of the structure. For large and complicated structures, there are generally six ways to obtain their transient responses:

1) Direct integration methods assume variation of velocity and acceleration, and they perform numerical integration directly on the physical finite element (FE) model of the structure. There is numerous research on these methods [1–6]. Among these methods, the central difference method and the Newmark's method, as described in [1,2], are two of the most widely used methods. These methods are of high accuracy, but they usually suffer a heavy computational burden; hence, they are computationally inefficient, especially for large-scale models.

2) The mode displacement superposition (MDS) method [7] is a technique that first transforms the physical FE model to the corresponding uncoupled modal model by using a subset of the mode shapes of the structure; it solves a series of modal equations of motion of a single degree of freedom (DOF), and then it calculates the physical response through a back transformation. The primary benefit of this method is the computational economy obtained by the mode truncation (i.e., model reduction) and the mode decoupling and, accordingly, analytical solutions for interpolated arbitrary excitation forces. However, this method can be applied only to proportionally damped structures.

3) The direct integration in the modal subspace [8] is a method for nonproportionally damped structures that implements direct integration methods, such as the central difference method and the Newmark's method, on the coupled modal equations of motion. The method retains the benefit of mode truncation, but it loses the advantage of mode decoupling and, consequently, the analytical solutions for interpolated arbitrary excitation forces. Nevertheless, this method is generally most welcome for solving large-scale engineering problems.

4) For nonproportionally damped structures, iterative methods [9–12] first decompose the coupled modal damping matrix into a

Received 11 November 2009; revision received 12 June 2010; accepted for publication 28 July 2010.

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diagonal part and a nondiagonal part, and they move the nondiagonal part multiplied by the velocity vector, called pseudoforce, to the right-hand side of the equations. They then solve the coupled equations of motion in an uncoupled way, which is the same way as the MDS method. Since the velocity is yet to be solved, this approach requires iterations to obtain solutions. Such requirements as convergence condition and high-convergence performance of the iterations often block the wide applications of these methods.

5) Perturbation methods [13,14] consider the damping of the damped structure as a perturbation of the corresponding undamped structure. The damped modes are then derived on the basis of undamped eigenpairs, and they are used to compute the dynamic response of the damped structure in a way similar to classical mode superposition. In addition to dynamic response, these methods provide physical insight of the damped modes of the structure. However, complex arithmetic is usually required in these procedures, since the damped modes are complex in nature.

6) Component mode synthesis (CMS) methods (see, for example, [15–22]) partition the system structure into several substructures (i.e., components) and represent each substructure by a set of suitably chosen modal and possibly physical coordinates and their associated mode shapes. The reduced-order substructure models are then assembled, yielding a set of synthesized differential equations of motion that must then be solved by direct integration methods or iterative methods for nonproportionally damped structures (or the MDS method for proportionally damped structures). The main motivation and advantage for these methods are the savings of computational costs and the capabilities of analyzing large-scale structures at their substructure level. In addition to the CMS methods, there are various component synthesis methods for transient response calculation. Gordis [23] presented a structural transient response synthesis method based on Volterra integral equations derived from the most basic convolution integral [24]. Although only physical coordinates of interest were retained, the associated submatrix of impulse response function, which is required in the integral equations, had to be calculated from the full-order substructure model if the substructure was nonproportionally damped. This process is inefficient for large-scale structures. Tsuei and Yee [25] developed a technique for calculating the transient response of structures that may be formed by joining components through connecting elements at the boundaries of the components. This technique is very efficient for recalculation of the transient response induced by modification of the connecting elements. However, only proportionally damped components are accepted for this method. Summarily, it can be said that these methods [23,25] are ineffective, inefficient, or both for nonproportionally damped structures.

In view of the fact that modern structures are often heavily nonproportionally damped due to the extensive applications of composite materials and/or various energy dissipation techniques [26–29], there are many studies that contribute to the dynamic response analysis of nonproportionally damped structures [30–32] in recent years. The motivation for the present work is to provide an effective and efficient transient response synthesis method for nonproportionally damped structures, especially for large-scale structures. The proposed method is an extension and an improvement of the method developed in [25]; that is, the studied structures may be formed by joining components through connecting elements at the boundaries. The extension indicates that nonproportionally damped components are admitted for the present method rather than only proportionally damped components accepted for the original method [25]. The improvement implies that piecewise linear interpolation of excitation forces are used for the calculation of substructure transient response rather than constant interpolation, which is required in the original method [25]. The present paper is organized as follows. A direct integration method is first developed in Sec. II to define transient response of nonproportionally damped substructures. The substructure partition and the substructure transient responses are presented in Sec. III. The synthesis procedure to obtain interface force and its excited responses is derived by using the displacement compatibility and interface force equilibrium equations in Sec. IV. Section V summarizes the usage of and remarks on the superiority of the synthesis method. Three numerical examples are given in Sec. VI to examine and illustrate the application of the synthesis method, and conclusions are made in Sec. VII.

II. Pseudoforce Mode Superposition Method

A. Brief Review of the Mode Superposition Method

The equation of motion represents a linear, damped structural system with n DOFs and can be written in a discrete form as

$$\begin{cases} \boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \tilde{\boldsymbol{C}}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = \boldsymbol{f}(t) \\ \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \dot{\boldsymbol{x}}(0) = \dot{\boldsymbol{x}}_0 \end{cases} \tag{1}$$

By carrying out an appropriate modal transformation for Eq. (1); that is,

$$\boldsymbol{x} = \boldsymbol{\Phi}\boldsymbol{q}, \qquad \dot{\boldsymbol{x}} = \boldsymbol{\Phi}\dot{\boldsymbol{q}}, \qquad \ddot{\boldsymbol{x}} = \boldsymbol{\Phi}\ddot{\boldsymbol{q}} \tag{2}$$

a set of uncoupled modal equations of motion is obtained for proportionally damped substructures as

$$\begin{cases} \bar{m}_r\ddot{q}_r(t) + \bar{c}_r\dot{q}_r(t) + \bar{k}_r q_r(t) = \bar{f}_r(t) \quad r = 1, 2, \ldots, m \\ q_r(0) = \boldsymbol{\varphi}_r^T\boldsymbol{M}\boldsymbol{x}_0/\bar{m}_r \\ \dot{q}_r(0) = \boldsymbol{\varphi}_r^T\boldsymbol{M}\dot{\boldsymbol{x}}_0/\bar{m}_r \end{cases} \tag{3}$$

With the use of the Duhamel's integral [8], the analytical solution of Eq. (3) can be written as

$$q_r(i+1) = a_r q_r(i) + b_r\dot{q}_r(i) + c_r\bar{f}_r(i) + d_r\bar{f}_r(i+1) \tag{4}$$

$$\dot{q}_r(i+1) = a'_r q_r(i) + b'_r\dot{q}_r(i) + c'_r\bar{f}_r(i) + d'_r\bar{f}_r(i+1) \tag{5}$$

$$\ddot{q}_r(i+1) = [\bar{f}_r(i+1) - \bar{c}_r\dot{q}_r(i+1) - \bar{k}_r q_r(i+1)]/m_r \tag{6}$$

where piecewise linear interpolation of excitation forces is assumed; that is, the real force is approximated as a first-order linear function between any two successive time points, as shown in Fig. 1. Also, a_r, b_r, c_r, d_r, a'_r, b'_r, c'_r, and d'_r are integral coefficients. The detailed expressions for these coefficients, as can be found in [8], are given in the Appendix. The i and $i+1$ denote the ith and $(i+1)$th time step, respectively, and $i = 0, 1, 2, \cdots$.

B. Proposed Pseudoforce Mode Superposition Method

In the case of nonproportionally damped substructures, the modal transformation equation (2) yields the following reduced and coupled modal system of $m \times m$:

$$\begin{cases} \bar{\boldsymbol{M}}\ddot{\boldsymbol{q}}(t) + (\bar{\boldsymbol{C}}_p + \bar{\boldsymbol{C}}_{np})\dot{\boldsymbol{q}}(t) + \bar{\boldsymbol{K}}\boldsymbol{q}(t) = \boldsymbol{\Phi}^T\boldsymbol{f}(t) \\ \boldsymbol{q}(0) = \bar{\boldsymbol{M}}^{-1}\boldsymbol{\Phi}^T\boldsymbol{M}\boldsymbol{x}_0 \\ \dot{\boldsymbol{q}}(0) = \bar{\boldsymbol{M}}^{-1}\boldsymbol{\Phi}^T\boldsymbol{M}\dot{\boldsymbol{x}}_0 \end{cases} \tag{7}$$

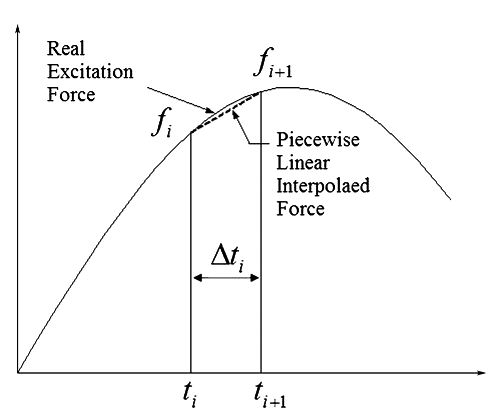


Fig. 1 Piecewise linear interpolation of the excitation force.

where $\bar{C}_p + \bar{C}_{np} = \Phi^T \tilde{C} \Phi$ is the modal damping matrix, and $\bar{C}_p$ and $\bar{C}_{np}$ are the diagonal and offdiagonal part of $\Phi^T \tilde{C} \Phi$, respectively.

To solve Eq. (7), the coupled terms $\bar{C}_{np}\dot{q}(t)$ are first treated as pseudoforces and moved to the right-hand side of Eq. (7):

$$\bar{M}\ddot{q}(t) + \bar{C}_p\dot{q}(t) + \bar{K}q(t) = \Phi^T f(t) - \bar{C}_{np}\dot{q}(t) \tag{8}$$

Then, Eq. (7) is turned into the same form of Eq. (3), regardless of the matrix form of Eq. (8). The solutions of Eq. (8) can be directly obtained by using the solution equations (4–6)

$$q(i+1) = Aq(i) + B\dot{q}(i) + C[\Phi^T f(i) - \bar{C}_{np}\dot{q}(i)] + D[\Phi^T f(i+1) - \bar{C}_{np}\dot{q}(i+1)] \tag{9}$$

$$\dot{q}(i+1) = A'q(i) + B'\dot{q}(i) + C'[\Phi^T f(i) - \bar{C}_{np}\dot{q}(i)] + D'[\Phi^T f(i+1) - \bar{C}_{np}\dot{q}(i+1)] \tag{10}$$

$$\ddot{q}(i+1) = \bar{M}^{-1}[\Phi^T f(i+1) - \bar{C}_{np}\dot{q}(i+1) - \bar{C}_p\dot{q}(i+1) - \bar{K}q(i+1)] \tag{11}$$

where A, B, C, D, A', B', C', and D' are diagonal integral coefficient matrices for which the diagonal elements are composed of a_r, b_r, c_r, d_r, a'_r, b'_r, c'_r, d'_r, and $r = 1, 2, \ldots, m$, respectively.

Because the right-hand side of Eq. (9) contains the unknown velocity response $\dot{q}(i+1)$, this equation cannot be used first to solve displacement response. However, it is found that the unknown velocity response $\dot{q}(i+1)$ can first be solved from Eq. (10); that is,

$$\dot{q}(i+1) = (I + D'\bar{C}_{np})^{-1}[A'q(i) + (B' - C'\bar{C}_{np})\dot{q}(i) + C'\Phi^T f(i) + D'\Phi^T f(i+1)] \tag{12}$$

By introducing the solution equation (12) to Eq. (9), the displacement response is obtained, which is

$$\begin{aligned} q(i+1) = {} & [A - D\bar{C}_{np}(I + D'\bar{C}_{np})^{-1}A']q(i) \\ & + [B - C\bar{C}_{np} + D\bar{C}_{np}(I + D'\bar{C}_{np})^{-1}(C'\bar{C}_{np} - B')]\dot{q}(i) \\ & + [C - D\bar{C}_{np}(I + D'\bar{C}_{np})^{-1}C']\Phi^T f(i) \\ & + [D - D\bar{C}_{np}(I + D'\bar{C}_{np})^{-1}D']\Phi^T f(i+1) \end{aligned} \tag{13}$$

Once the displacement and velocity are known, the acceleration can be solved from Eq. (11). It can be seen that the just-developed procedure directly solves the nonproportionally damped system by using the concept of pseudoforce of iterative methods and the analytical solution of the MDS method in the sense of piecewise linear interpolation of excitation forces; hence, it is called the pseudoforce MDS (PFMDS) method in this paper. Compared with the traditional direct methods and iterative methods for solving a nonproportionally damped system, while avoiding iterations, the PFMDS method keeps the advantage of the analytical solution of the MDS method for linear interpolated excitation forces. The PFMDS method will be used to define the transient response of nonproportionally damped substructures in Sec. III. However, it should be pointed out that the PFMDS method is applicable to general transient response calculations of nonproportionally damped structures.

It is relevant to note that an explicit matrix inversion, i.e., $(I + D'\bar{C}_{np})^{-1}$, is required in the PFMDS method. This matrix is of the size defined by the number of modes involved for the calculation in the modal space, which may be hundreds to thousands, but is often 10s to 100s, even for large-scale structural engineering problems; thus, the inversion can be easily handled by modern-day moderate computers. Also, the inversion has to be calculated only once for a constant increment of the integration time step. Another observation for the PFMDS method is that, when the substructure is proportionally damped, $\bar{C}_{np} = 0$, and the PFMDS method automatically reduces to the MDS method.

For ease of notation, Eqs. (12) and (13) are, respectively, rewritten as

$$q(i+1) = Eq(i) + F\dot{q}(i) + G\Phi^T f(i) + H\Phi^T f(i+1) \tag{14}$$

and

$$\dot{q}(i+1) = E'q(i) + F'\dot{q}(i) + G'\Phi^T f(i) + H'\Phi^T f(i+1) \tag{15}$$

where $E' = (I + D'\bar{C}_{np})^{-1}A'$, $E = A - D\bar{C}_{np}E'$, $F' = (I + D'\bar{C}_{np})^{-1}(B' - C'\bar{C}_{np})$, $F = B - C\bar{C}_{np} - D\bar{C}_{np}F'$, $G' = (I + D'\bar{C}_{np})^{-1}C'$, $G = C - D\bar{C}_{np}G'$, $H' = (I + D'\bar{C}_{np})^{-1}D'$, and $H = D - D\bar{C}_{np}H'$.

III. Substructure Partition and Substructure Transient Response

Consider a simple but representative structure composed of two substructures joined by one connecting element, as shown in Fig. 2. According to the geometric connecting topology, the system structure is partitioned into two substructures, as shown in Fig. 3. After the partition, the DOFs of each substructure are partitioned into interface DOFs and internal DOFs; consequently, the response vectors of the substructures can be written as

$$x^\alpha = \begin{Bmatrix} x_j^\alpha \\ x_i^\alpha \end{Bmatrix}, \qquad x^\beta = \begin{Bmatrix} x_j^\beta \\ x_i^\beta \end{Bmatrix} \tag{16}$$

After the partition, the connecting forces between substructures (i.e., the interface forces) are exposed; then the responses of each substructure can be decomposed into two parts: responses excited by external forces with predefined initial conditions and responses excited by interface forces with zero initial conditions. Thus, the responses of each substructure can be written as

$$q(i+1) = q_e(i+1) + q_j(i+1) \tag{17}$$

$$\dot{q}(i+1) = \dot{q}_e(i+1) + \dot{q}_j(i+1) \tag{18}$$

$$\ddot{q}(i+1) = \ddot{q}_e(i+1) + \ddot{q}_j(i+1) \tag{19}$$

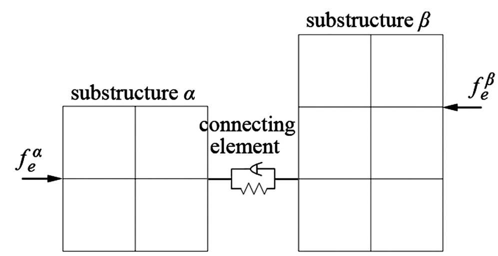


Fig. 2 The system structure.

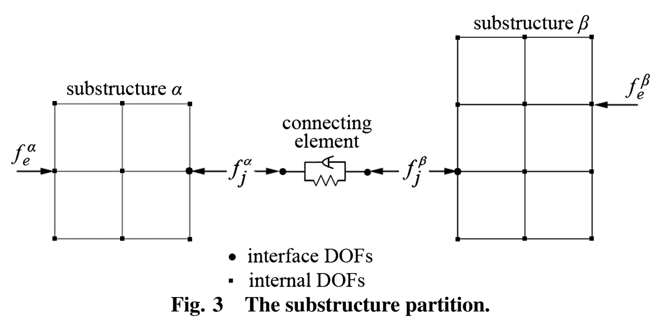


Fig. 3 The substructure partition.

where the subscripts e and j denote the response component excited by external and interface forces, respectively.

By applying the PFMDS method [i.e., Eqs. (11), (14), and (15)] to each substructure, the response components can be defined as

$$\begin{cases} \boldsymbol{q}_e(i+1) = \boldsymbol{E}\boldsymbol{q}_e(i) + \boldsymbol{F}\dot{\boldsymbol{q}}_e(i) + \boldsymbol{G}\boldsymbol{\Phi}^T\boldsymbol{f}_e(i) + \boldsymbol{H}\boldsymbol{\Phi}^T\boldsymbol{f}_e(i+1) \\ \dot{\boldsymbol{q}}_e(i+1) = \boldsymbol{E}'\boldsymbol{q}_e(i) + \boldsymbol{F}'\dot{\boldsymbol{q}}_e(i) + \boldsymbol{G}'\boldsymbol{\Phi}^T\boldsymbol{f}_e(i) + \boldsymbol{H}'\boldsymbol{\Phi}^T\boldsymbol{f}_e(i+1) \\ \ddot{\boldsymbol{q}}_e(i+1) = \bar{\boldsymbol{M}}^{-1}[\boldsymbol{\Phi}^T\boldsymbol{f}_e(i+1) - (\bar{\boldsymbol{C}}_p + \bar{\boldsymbol{C}}_{np})\dot{\boldsymbol{q}}_e(i+1) - \boldsymbol{\Lambda}\boldsymbol{q}_e(i+1)] \\ \boldsymbol{q}_e(0) = \bar{\boldsymbol{M}}^{-1}\boldsymbol{\Phi}^T\boldsymbol{M}\boldsymbol{x}_0 \\ \dot{\boldsymbol{q}}_e(0) = \bar{\boldsymbol{M}}^{-1}\boldsymbol{\Phi}^T\boldsymbol{M}\dot{\boldsymbol{x}}_0 \end{cases} \tag{20}$$

and

$$\begin{cases} \boldsymbol{q}_j(i+1) = \boldsymbol{E}\boldsymbol{q}_j(i) + \boldsymbol{F}\dot{\boldsymbol{q}}_j(i) + \boldsymbol{G}\boldsymbol{\Phi}_j^T\boldsymbol{f}_j(i) + \boldsymbol{H}\boldsymbol{\Phi}_j^T\boldsymbol{f}_j(i+1) \\ \dot{\boldsymbol{q}}_j(i+1) = \boldsymbol{E}'\boldsymbol{q}_j(i) + \boldsymbol{F}'\dot{\boldsymbol{q}}_j(i) + \boldsymbol{G}'\boldsymbol{\Phi}_j^T\boldsymbol{f}_j(i) + \boldsymbol{H}'\boldsymbol{\Phi}_j^T\boldsymbol{f}_j(i+1) \\ \ddot{\boldsymbol{q}}_j(i+1) = \bar{\boldsymbol{M}}^{-1}[\boldsymbol{\Phi}_j^T\boldsymbol{f}_j(i+1) - (\bar{\boldsymbol{C}}_p + \bar{\boldsymbol{C}}_{np})\dot{\boldsymbol{q}}_j(i+1) - \boldsymbol{\Lambda}\boldsymbol{q}_j(i+1)] \\ \boldsymbol{q}_j(0) = 0 \\ \dot{\boldsymbol{q}}_j(0) = 0 \end{cases} \tag{21}$$

The response $\boldsymbol{q}_e$ and its derivatives are immediately determined from Eq. (20), since the external forces $\boldsymbol{f}_e$ and the initial conditions $\boldsymbol{x}_0$ and $\dot{\boldsymbol{x}}_0$ are predefined. However, the response $\boldsymbol{q}_j$ and its derivatives cannot be solved from Eq. (21) yet, as the interface forces $\boldsymbol{f}_j$ are yet unknown. These quantities will be determined together in the substructure synthesis process in Sec. IV.

IV. Synthesis of Whole System Response

The whole structure response is obtained from the synthesis of substructures, and the synthesis is achieved by using the displacement compatibility condition between substructures and connecting elements as well as the interface force equilibrium condition of the connecting elements. The displacement compatibility condition is

$$\boldsymbol{x}_j^\alpha = \boldsymbol{x}_c^\alpha, \qquad \boldsymbol{x}_j^\beta = \boldsymbol{x}_c^\beta \tag{22}$$

where $\boldsymbol{x}_c^\alpha$ and $\boldsymbol{x}_c^\beta$ are the displacement of the connecting element adjacent to the substructures α and β, respectively.

The force equilibrium condition is

$$\boldsymbol{f}_j^\alpha = -\boldsymbol{f}_j^\beta \tag{23}$$

where the interface force is defined as

$$\boldsymbol{f}_j^\alpha = \boldsymbol{K}_c(\boldsymbol{x}_c^\beta - \boldsymbol{x}_c^\alpha) + \boldsymbol{C}_c(\dot{\boldsymbol{x}}_c^\beta - \dot{\boldsymbol{x}}_c^\alpha) \tag{24}$$

By introducing the displacement compatibility condition, Eqs. (22) and (24) can be rewritten as

$$\boldsymbol{f}_j^\alpha = \boldsymbol{K}_c(\boldsymbol{x}_j^\beta - \boldsymbol{x}_j^\alpha) + \boldsymbol{C}_c(\dot{\boldsymbol{x}}_j^\beta - \dot{\boldsymbol{x}}_j^\alpha) \tag{25}$$

According to the modal transformation equation (2), we have

$$\begin{aligned} \boldsymbol{x}_j^\alpha(i+1) &= \boldsymbol{\Phi}_j^\alpha\boldsymbol{q}^\alpha(i+1), \qquad \dot{\boldsymbol{x}}_j^\alpha(i+1) = \boldsymbol{\Phi}_j^\alpha\dot{\boldsymbol{q}}^\alpha(i+1) \\ \boldsymbol{x}_j^\beta(i+1) &= \boldsymbol{\Phi}_j^\beta\boldsymbol{q}^\beta(i+1), \qquad \dot{\boldsymbol{x}}_j^\beta(i+1) = \boldsymbol{\Phi}_j^\beta\dot{\boldsymbol{q}}^\beta(i+1) \end{aligned} \tag{26}$$

By introducing Eq. (26) to Eq. (25) and recognizing that the responses are decomposed into two parts, as shown in Eqs. (17) and (18), the interface force is then expressed as

$$\begin{aligned} \boldsymbol{f}_j^\alpha(i+1) = {} & \boldsymbol{K}_c\{\boldsymbol{\Phi}_j^\beta[\boldsymbol{q}_e^\beta(i+1) + \boldsymbol{q}_j^\beta(i+1)] - \boldsymbol{\Phi}_j^\alpha[\boldsymbol{q}_e^\alpha(i+1) \\ & + \boldsymbol{q}_j^\alpha(i+1)]\} + \boldsymbol{C}_c\{\boldsymbol{\Phi}_j^\beta[\dot{\boldsymbol{q}}_e^\beta(i+1) + \dot{\boldsymbol{q}}_j^\beta(i+1)] \\ & - \boldsymbol{\Phi}_j^\alpha[\dot{\boldsymbol{q}}_e^\alpha(i+1) + \dot{\boldsymbol{q}}_j^\alpha(i+1)]\} \end{aligned} \tag{27}$$

Substituting the responses defined by Eq. (21) for $\boldsymbol{q}_j^\beta(i+1)$ and $\boldsymbol{q}_j^\alpha(i+1)$ in Eq. (27) leads to

$$\begin{aligned} \boldsymbol{f}_j^\alpha(i+1) = {} & \boldsymbol{K}_c\boldsymbol{\Phi}_j^\beta[\boldsymbol{q}_e^\beta(i+1) + \boldsymbol{E}^\beta\boldsymbol{q}_j^\beta(i) + \boldsymbol{F}^\beta\dot{\boldsymbol{q}}_j^\beta(i) \\ & + \boldsymbol{G}^\beta\boldsymbol{\Phi}_j^{\beta T}\boldsymbol{f}_j^\beta(i) + \boldsymbol{H}^\beta\boldsymbol{\Phi}_j^{\beta T}\boldsymbol{f}_j^\beta(i+1)] - \boldsymbol{K}_c\boldsymbol{\Phi}_j^\alpha[\boldsymbol{q}_e^\alpha(i+1) \\ & + \boldsymbol{E}^\alpha\boldsymbol{q}_j^\alpha(i) + \boldsymbol{F}^\alpha\dot{\boldsymbol{q}}_j^\alpha(i) + \boldsymbol{G}^\alpha\boldsymbol{\Phi}_j^{\alpha T}\boldsymbol{f}_j^\alpha(i) + \boldsymbol{H}^\alpha\boldsymbol{\Phi}_j^{\alpha T}\boldsymbol{f}_j^\alpha(i+1)] \\ & + \boldsymbol{C}_c\boldsymbol{\Phi}_j^\beta[\dot{\boldsymbol{q}}_e^\beta(i+1) + \boldsymbol{E}'^\beta\boldsymbol{q}_j^\beta(i) + \boldsymbol{F}'^\beta\dot{\boldsymbol{q}}_j^\beta(i) + \boldsymbol{G}'^\beta\boldsymbol{\Phi}_j^{\beta T}\boldsymbol{f}_j^\beta(i) \\ & + \boldsymbol{H}'^\beta\boldsymbol{\Phi}_j^{\beta T}\boldsymbol{f}_j^\beta(i+1)] - \boldsymbol{C}_c\boldsymbol{\Phi}_j^\alpha[\dot{\boldsymbol{q}}_e^\alpha(i+1) + \boldsymbol{E}'^\alpha\boldsymbol{q}_j^\alpha(i) \\ & + \boldsymbol{F}'^\alpha\dot{\boldsymbol{q}}_j^\alpha(i) + \boldsymbol{G}'^\alpha\boldsymbol{\Phi}_j^{\alpha T}\boldsymbol{f}_j^\alpha(i) + \boldsymbol{H}'^\alpha\boldsymbol{\Phi}_j^{\alpha T}\boldsymbol{f}_j^\alpha(i+1)] \end{aligned} \tag{28}$$

By using the force equilibrium condition equation (23), the following equation is finally derived:

$$\begin{aligned} \boldsymbol{I}_c^*\boldsymbol{f}_j^\alpha(i+1) = {} & \boldsymbol{K}_c\boldsymbol{\Phi}_j^\beta[\boldsymbol{q}_e^\beta(i+1) + \boldsymbol{E}^\beta\boldsymbol{q}_j^\beta(i) + \boldsymbol{F}^\beta\dot{\boldsymbol{q}}_j^\beta(i) \\ & - \boldsymbol{G}^\beta\boldsymbol{\Phi}_j^{\beta T}\boldsymbol{f}_j(i)] - \boldsymbol{K}_c\boldsymbol{\Phi}_j^\alpha[\boldsymbol{q}_e^\alpha(i+1) + \boldsymbol{E}^\alpha\boldsymbol{q}_j^\alpha(i) + \boldsymbol{F}^\alpha\dot{\boldsymbol{q}}_j^\alpha(i) \\ & + \boldsymbol{G}^\alpha\boldsymbol{\Phi}_j^{\alpha T}\boldsymbol{f}_j(i)] + \boldsymbol{C}_c\boldsymbol{\Phi}_j^\beta[\dot{\boldsymbol{q}}_e^\beta(i+1) + \boldsymbol{E}'^\beta\boldsymbol{q}_j^\beta(i) + \boldsymbol{F}'^\beta\dot{\boldsymbol{q}}_j^\beta(i) \\ & - \boldsymbol{G}'^\beta\boldsymbol{\Phi}_j^{\beta T}\boldsymbol{f}_j(i)] - \boldsymbol{C}_c\boldsymbol{\Phi}_j^\alpha[\dot{\boldsymbol{q}}_e^\alpha(i+1) + \boldsymbol{E}'^\alpha\boldsymbol{q}_j^\alpha(i) + \boldsymbol{F}'^\alpha\dot{\boldsymbol{q}}_j^\alpha(i) \\ & + \boldsymbol{G}'^\alpha\boldsymbol{\Phi}_j^{\alpha T}\boldsymbol{f}_j(i)] \end{aligned} \tag{29}$$

where

$$\begin{aligned} \boldsymbol{I}_c^* = {} & \boldsymbol{I} + \boldsymbol{K}_c\boldsymbol{\Phi}_j^\beta\boldsymbol{H}^\beta\boldsymbol{\Phi}_j^{\beta T} + \boldsymbol{K}_c\boldsymbol{\Phi}_j^\alpha\boldsymbol{H}^\alpha\boldsymbol{\Phi}_j^{\alpha T} \\ & + \boldsymbol{C}_c\boldsymbol{\Phi}_j^\beta\boldsymbol{H}'^\beta\boldsymbol{\Phi}_j^{\beta T} + \boldsymbol{C}_c\boldsymbol{\Phi}_j^\alpha\boldsymbol{H}'^\alpha\boldsymbol{\Phi}_j^{\alpha T} \end{aligned}$$

Equation (29) sets a recursive formula to calculate the interface forces required in Eq. (21) to solve $\boldsymbol{q}_j$. The initial value for this equation is determined based on the initial conditions of the whole structure as

$$\boldsymbol{f}_j^\alpha(0) = \boldsymbol{K}_c[\boldsymbol{x}_j^\beta(0) - \boldsymbol{x}_j^\alpha(0)] + \boldsymbol{C}_c[\dot{\boldsymbol{x}}_j^\beta(0) - \dot{\boldsymbol{x}}_j^\alpha(0)] \tag{30}$$

Because the right-hand side of Eq. (29) contains the responses $\boldsymbol{q}_j$ and $\dot{\boldsymbol{q}}_j$, the calculations of the interface forces $\boldsymbol{f}_j^\alpha$ and its excited responses are a synchronous process; this process is achieved by performing recursive calculation of Eqs. (21) and (29). Once the

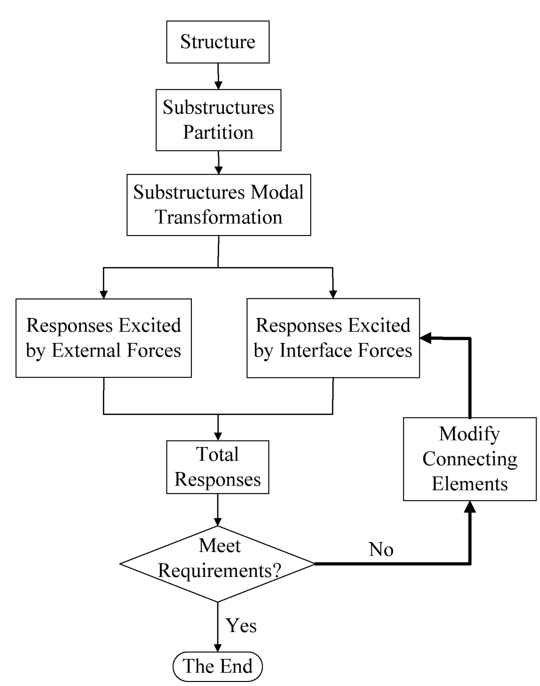


Fig. 4 Flowchart of the synthesis method.

Table 1 Comparison of operation counts for one recalculation[a]

Step	Task	MCD method in modal subspace Cost	Synthesis method Cost
1	Eigenvalue analysis	$O(n \times b^2 \times m)$ [Lanczos method]	Not required
2	Modal solution	$O(m^3) + O(m^2 \times N_t)$	$O(j^3) + O(m^2 \times N_t)$
3	Physical solution	$O(n \times m \times N_t)$	$O(n \times m \times N_t)$

[a]The n denotes the number of physical DOFs, b indicates semibandwidth of the eigenvalue problem, m denotes the number of kept modes, j denotes the number of interface DOFs, and N_t is the number of time steps in the integration.

interface-force-excited responses are obtained, the total responses can be obtained from Eqs. (17–19), and then a back modal transformation [i.e., Eq. (2)] can be carried out to obtain physical responses.

V. Summary of Method

A transient response synthesis method for structures composed of nonproportionally damped components joined by connecting elements is developed in the preceding sections. The method can be summarized as follows:

1) For a given structure, partition it into several substructures according to the geometric connecting topology. The dynamic motion of each substructure is described by Eq. (1).

2) Perform a partial eigensolution for each substructure and transform Eq. (1) into modal form [i.e., Eq. (7)] by using a subset of modes of the substructure.

3) Calculate the modal responses excited by external forces for each substructure by using Eq. (20) and transform the modal responses to physical responses by using Eq. (2).

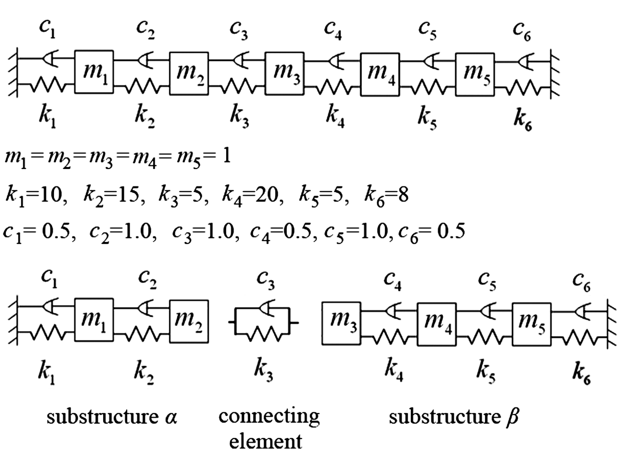


Fig. 5 The lumped mass-spring-damper system and its substructures.

4) Calculate the modal responses excited by interface forces for each substructure by using Eqs. (21), (29), and (30) and transform the modal responses to physical responses by using Eq. (2).

5) Calculate the total responses by adding the response components obtained in steps 3 and 4 together.

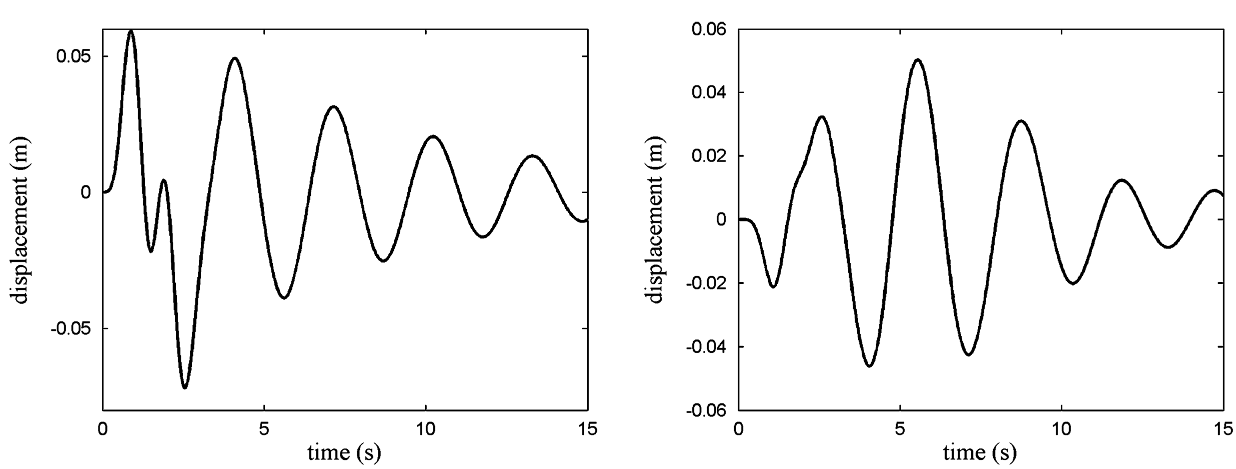
According to the procedure just discussed, the most remarkable superiority of the proposed synthesis method is that the method provides an efficient algorithm when the connecting elements are undergoing frequent modification during the design of the whole structure because, in a such modification, only responses excited by interface forces need to be recalculated, and substructure modes remain unchanged. Also, modes of the whole structure are not required; that is, only steps 4 and 5 need to be reperformed. This recalculation process is different from that of general CMS methods; the recalculation of the substructure responses of the proposed method are directly performed based on the modified interface force equation [i.e., Eq. (29)]. This process is clearly shown in Fig. 4 as a flowchart.

Table 1 compares the main number of operations of one recalculation for the MCD method in modal subspace and the proposed method. It can be seen from Table 1 that the proposed method needs two steps to perform the recalculation, while the general MCD method in modal subspace requires three steps. Besides the eigenvalue analysis, the main difference occurs in step 2; that is, $O(j^3) + O(m^2 \times N_t)$ and $O(m^3) + O(m^2 \times N_t)$ operations to compute the modal solution are required for the synthesis method and the MCD method, respectively. This difference implies that the modal solution step of the proposed method achieves higher efficiency than the MCD method when the number of interface DOFs is fewer than the number of kept modes.

VI. Numerical Examples

A. Lumped Mass-Spring-Damper Model

To validate the correctness of the proposed method, a five-DOF lumped mass-spring-damper model, as shown in Fig. 5, is first selected as a test model. The model is nonproportionally damped in either system or component level. The spring k_3 and damper c_3 are



a) Excited by external force **b) Excited by interface force**

Fig. 6 The displacement response components of mass m_2.

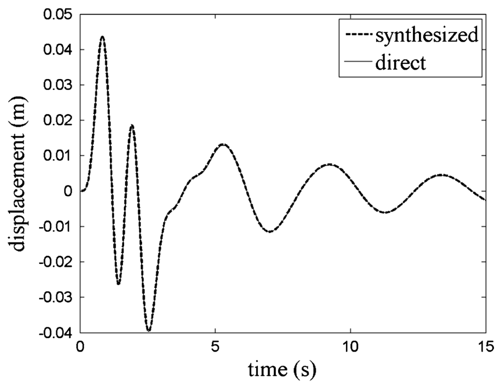


Fig. 7 The total displacement response of mass m_2.

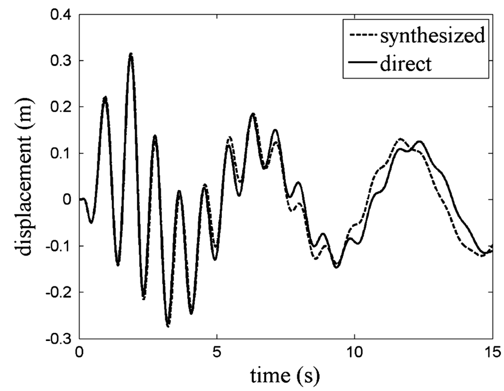


Fig. 9 The displacement response of node 41 in the Y direction.

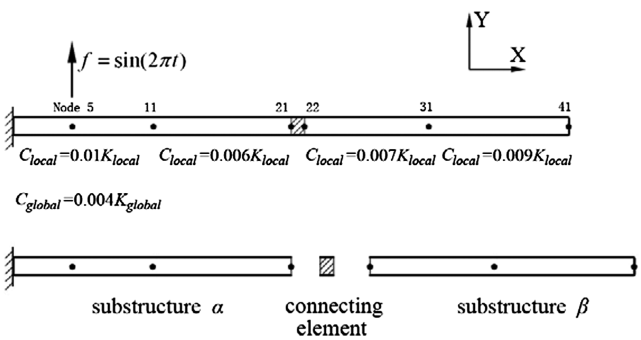


Fig. 8 A damped cantilever beam and its substructures.

used to connect substructures α and β. A force $f_e = \sin(2\pi t)$ is applied on location m_1, and the action time of the force is 0.0–2.0 s. The proposed method and the modified central difference (MCD) method [6] are used to calculate the transient responses of the model. Piecewise linear interpolation of the force is assumed in the numerical calculation. Figure 6 shows the displacement response components of mass m_2 excited by external and interface forces. The addition of theses two response components leads to the total response (i.e., the synthesized solution), as shown in Fig. 7. It can be seen from Fig. 7 that the synthesized response is exactly identical to the direct solution with respect to the resolution of the plot. This can prove the correctness of the proposed synthesis method.

B. Damped Cantilever Beam Model

A damped cantilever beam, as shown in Fig. 8, is studied to further examine the synthesis method. The beam is made of aluminum with a length of 4.0 m, and it has a circular section with a radius of 2.0 mm. The used material properties of the aluminum are Young's modulus of 70 GPa, Poisson's ratio of 0.3, and density of 2700 $\mathrm{Kg/m^3}$. The structure is discretized evenly with 40 elements (i.e., 41 nodes), and it is divided into two components, α and β, and they are connected by a massless beam element. Each node has three DOFs: an X-translation, a Y-translation, and a Z-rotation DOF (the X, Y, and Z axes compose a right-hand coordinate system). Local and global stiffness-proportional damping are assigned, as shown in Fig. 8, so that the beam is nonproportionally damped in both system and component levels; that is, $C_{\mathrm{local}} = 0.01K_{\mathrm{local}}$ for elements 1 to 10, $C_{\mathrm{local}} = 0.006K_{\mathrm{local}}$ for elements 11 to 20, $C_{\mathrm{local}} = 0.007K_{\mathrm{local}}$ for elements 22 to 30, $C_{\mathrm{local}} = 0.009K_{\mathrm{local}}$ for elements 31 to 40, and $C_{\mathrm{global}} = 0.004K_{\mathrm{global}}$ for all of the 40 elements. A piecewise linear interpolated force of $f_e = \sin(2\pi t)$ is exerted on node 5 in the Y direction, and the action time of the force is 0.0–2.0 s. After the partition, the substructure α is still a cantilever beam, and the substructure β is a

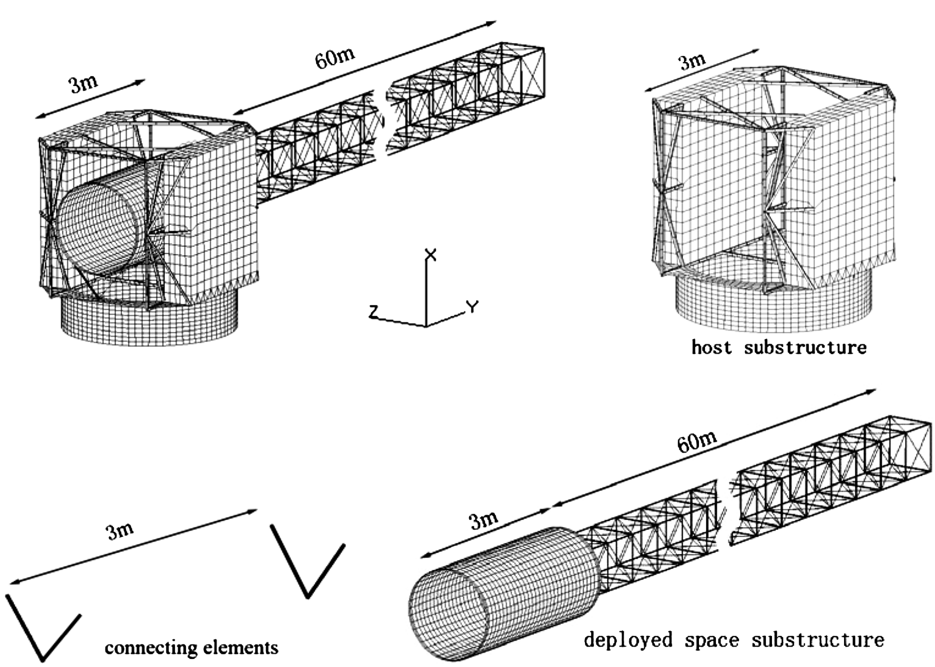


Fig. 10 The large deployed space structure and its substructures.

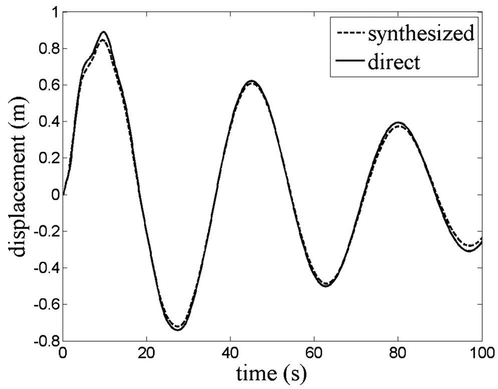


Fig. 11 The displacement response at the right tip of the deployed mast.

free–free beam. The first 15 substructure modes for each substructure are included for the synthesis. The synthesized solution is compared with that of the MCD method, as shown in Fig. 9, for the Y-displacement response of node 41. It can be seen from the comparison that the synthesized solution agrees well with the direct solution from 0.0 to 7.0 s; however, minor differences are observed after 7.0 s. This difference is mainly induced by the truncation of higher modes of substructures and accumulated errors.

C. Large Deployed Space Structure

This subsection presents an engineering application of the synthesis method in structure modification. The studied structure is shown in Fig. 10. It is composed of a host substructure and a deployed space substructure, and the substructures are connected by four massless damped beam elements. Both substructures are modeled with beam and shell elements, and they are nonproportionally damped due to the use of various materials with different damping coefficients for different types of elements; that is, the damping coefficients of the beam and shell elements are assumed to be 0.015 and 0.02, respectively. To discretize the structure, 32,772 DOFs are used. The connecting elements intensively affect the dynamic responses of the structure, and they are of importance in the primary design stage of the structure; thus, their parameters need to be frequently modified. Hence, the synthesis method is used for the simulation design of this structure. To simulate the dynamic environment that the structure suffers, the enforced motion of pulse displacement in the X direction at the bottom of the host substructure is applied. The pulse is a triangular wave with a duration of 0.2 s and a magnitude of 0.01 m. For this excitation, the assumed piecewise linear interpolation is accurate. The first 10 modes of each substructure are used for the synthesis method. The displacement response at the right tip of the deployed mast is calculated and compared with the direct solution of the MCD method, as shown in Fig. 11 for accuracy verification. It can be observed that the synthesized method achieves satisfactory results. Because of the large flexibility of the structure, the truncation of higher modes does not induce obvious errors, as observed in the beam example.

According to the discussion in Sec. V, when the connecting elements undergo frequent modification, the synthesis method offers an efficient computation tool. Table 2 shows the elapsed time of one recalculation for the synthesis method and the MCD method in modal subspace. For the MCD method in modal subspace, the first 10 modes of the structure are retained. According to the comparison, the efficiency of the synthesis method is prominently high compared with that of the MCD method in the modal subspace.

Table 2 Elapsed time of one recalculation

Method	Elapsed time, s
MCD method in modal subspace	6.832
Synthesis method	0.625

VII. Conclusions

A synthesis method for calculating the transient responses of nonproportionally damped systems is presented. Its characters and advantages can be summarized as follows:

1) The method uses the proposed PFMDS procedure to define the transient responses of nonproportionally damped components. The PFMDS solves the nonproportionally damped modal equation directly and keeps the analytical solutions of the MDS method for interpolated excitation forces.

2) The method is based on modal parameters of components. Hence, a mode truncation can be taken to reduce the size of the computed model. However, the subset of kept modes should be chosen carefully so that the truncated modes do not induce unaccepted errors.

3) Unlike a traditional CMS or substructuring approach, the synthesized equation of the proposed method is the interface force equation [i.e., Eq. (29)]. This equation is directly used to recover the responses of the substructures; therefore, the eigensolution and the dynamic response solution of the synthesized system of the traditional CMS procedure are not required.

4) The method shows an outstanding computational performance in the recalculation of whole structure responses induced by modification of the connecting elements. Hence, the method is readily applied for structure modification problems and parametric studies.

5) When the components composing the nonproportionally damped structure are proportionally damped, the method reduced the nonproportionally damped problem to a proportionally damped one.

Three numerical examples are given to validate the proposed method. Numerical results indicate that the suggested method can obtain satisfactory results compared with that of the direct integration method.

Appendix

The coefficients a_r, b_r, c_r, d_r, a'_r, b'_r, c'_r, and d'_r defined in Eqs. (4) and (5) are defined as (for convenience, the subscript r denoting the rth mode, is omitted in the rest of the Appendix)

$$a = e^{-\xi\omega_n\Delta t}\left[\frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_d\Delta t) + \cos(\omega_d\Delta t)\right]$$

$$b = e^{-\xi\omega_n\Delta t}\left[\frac{1}{\omega_d}\sin(\omega_d\Delta t)\right]$$

$$c = \frac{1}{k}\left\{\frac{2\xi}{\omega_n\Delta t} + e^{-\xi\omega_n\Delta t}\left[\left(\frac{1-2\xi^2}{\omega_d\Delta t} - \frac{\xi}{\sqrt{1-\xi^2}}\right)\sin(\omega_d\Delta t) - \left(1+\frac{2\xi}{\omega_n\Delta t}\right)\cos(\omega_d\Delta t)\right]\right\}$$

$$d = \frac{1}{k}\left\{1 - \frac{2\xi}{\omega_n\Delta t} + e^{-\xi\omega_n\Delta t}\left[\frac{2\xi^2-1}{\omega_d\Delta t}\sin(\omega_d\Delta t) + \frac{2\xi}{\omega_n\Delta t}\cos(\omega_d\Delta t)\right]\right\}$$

$$a' = -e^{-\xi\omega_n\Delta t}\left[\frac{\omega_n}{\sqrt{1-\xi^2}}\sin(\omega_d\Delta t)\right]$$

$$b' = e^{-\xi\omega_n\Delta t}\left[\cos(\omega_d\Delta t) - \frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_d\Delta t)\right]$$

$$c' = \frac{1}{k}\left\{-\frac{1}{\Delta t} + e^{-\xi\omega_n\Delta t}\left[\left(\frac{\omega_n}{\sqrt{1-\xi^2}} + \frac{\xi}{\Delta t\sqrt{1-\xi^2}}\right)\sin(\omega_d\Delta t) + \frac{1}{\Delta t}\cos(\omega_d\Delta t)\right]\right\}$$

$$d' = \frac{1}{k\Delta t}\left\{1 - e^{-\xi\omega_n\Delta t}\left[\frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_d\Delta t) + \cos(\omega_d\Delta t)\right]\right\}$$

where ξ, ω_n, and ω_d indicate the rth modal damping coefficient, modal frequency, and damped modal frequency, respectively, and Δt denotes the time increment of the numerical integration.

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